



首都师范大学数学科学学院
School of Mathematical Sciences Capital Normal University



交叉科学研究院
ACADEMY FOR MULTIDISCIPLINARY STUDIES

2022 年首都师范大学数论研讨会

会议手册

首都师范大学数学科学学院、交叉科学研究院
北京国家应用数学中心
2022 年 4 月 7-8 日 北京

2022 年首都师范大学数论研讨会

2022 年 4 月 7 日至 4 月 8 日，首都师范大学数学科学学院与交叉科学研究院将联合主办数论研讨会，本次研讨会邀请国内外杰出学者，就数论及相关研究的最新动态和前沿信息进行深入探讨，交流最新的研究成果和应用进展，促进多学科交叉研究，推进“双一流”学科建设。

学术委员会（按姓氏排序）：

冯克勤（清华大学）

扶 磊（清华大学）

刘建亚（山东大学）

秦厚荣（南京大学）

田 野（中国科学院晨兴数学中心）

万大庆（美国加州大学欧文分校）

徐 飞（首都师范大学）

会议组委会：陈红星、方江学、唐舜

童纪龙、张 俊、赵斌

首都师范大学数学科学学院

首都师范大学交叉科学研究院

北京国家应用数学中心

会议日程

4月7日上午 腾讯会议号: 187 768 960			
8:50-9:00		致欢迎词	
时间	报告人	报告题目	主持人
9:00-9:45	秦厚荣	Vandiver 猜想与整数环的 K 理论	冯克勤
9:45-9:50	休息		
9:50-10:35	万大庆	Divisibility of Frobenius eigenvalues on ℓ -adic cohomology	徐 飞
10:35-10:40	休息		
10:40-11:25	洪绍方	Counting rational points on the hypersurface $f(x_1) + \dots + f(x_n) = a$ over finite fields	
4月7日下午 教二楼 610 (线下), 腾讯会议号: 187 768 960 (线上)			
14:00-14:45	田 野	L -values of elliptic curves and ternary quadratic forms	童纪龙
14:45-14:50	休息		
14:50-15:35	扶 磊	A p -adic Landau-Ginzburg B-model	方江学
15:35-15:40	休息		
15:40-16:25	肖 梁	模形式 p 进斜率与 Bergdall-Pollack 幽灵猜想	

4月8日 腾讯会议号: 187 768 960			
时间	报告人	报告题目	主持人
9:00-9:45	欧阳毅	The growth of Tate-Shafarevich groups in $\mathbb{Z}/p\mathbb{Z}$ -extensions	赵 斌
9:45-9:50	休息		
9:50-10:35	岳 勤	Weil 指数和与 Kloosterman 和性质以及在编码运用	张 俊
10:35-10:40	休息		
10:40-11:25	向 青	Constructions of m -ovoids of the symplectic polar spaces	
11:25-14:00 午休			
14:00-14:45	徐哲峰	Distribution of powers in the ring \mathbb{Z}_q	张文鹏
14:45-14:50	休息		
14:50-15:35	翟 帅	On the 2-part of the Birch-Swinnerton-Dyer exact formula	唐 舜
15:35-15:40	休息		
15:40-16:25	黄炳荣	L -函数的亚凸性界	

报告摘要

Speaker: 秦厚荣 (南京大学)

Title: Vandiver 猜想与整数环的 K 理论

Abstract: 假设 p 是一个奇素数, $K = \mathbb{Q}(\zeta_p)$ 为 p 次分圆域, K^+ 为其极大实子域。记 h_p^+ 为 K^+ 的类数。Vandiver 猜想 (Kummer-Vandiver 猜想) p 不整除 h_p^+ 。报告介绍这一猜想与整数环的 K 群的关系; 介绍已有的研究, 包括我们最近的研究成果。

Speaker: 万大庆 (美国加州大学欧文分校)

Title: Divisibility of Frobenius Eigenvalues on ℓ -adic Cohomology

Abstract: For an affine variety defined over a finite field with q elements, it is shown that as algebraic integers, the Frobenius eigenvalues on ℓ -adic cohomology have higher than known q -divisibility beyond the middle dimension. This sharpens both Deligne's integrality theorem (1973) and Esnault-Katz's cohomological divisibility theorem (2005).

Similar lower bounds are proved for the Hodge level for a complex variety, improving earlier results in this direction. This is joint work with Helene Esnault.

Speaker: 洪绍方 (四川大学)

Title: Counting rational points on the hypersurface $f(x_1) + \dots + f(x_n) = a$ over finite fields

Abstract: Let p be a prime, k a positive integer and let \mathbb{F}_q be the finite field of $q = p^k$ elements. Let $f(x)$ be a polynomial over \mathbb{F}_q and $a \in \mathbb{F}_q$. We denote by $N_s(f, a)$ the number of zeros $(x_1, \dots, x_s) \in \mathbb{F}_q^s$ of $f(x_1) + \dots + f(x_s) = a$. In this talk, we show that

$$\sum_{s=1}^{\infty} N_s(f, 0)t^s = \frac{t}{1-qt} - \frac{tM_f'(t)}{qM_f(t)},$$

where

$$M_f(t) := \prod_{\substack{m \in \mathbb{F}_q^* \\ S_{f,m} \neq 0}} \left(t - \frac{1}{S_{f,m}} \right)$$

with $S_{f,m} := \sum_{x \in \mathbb{F}_q} \zeta_p^{\text{Tr}(mf(x))}$, ζ_p being the p -th primitive unit root and Tr being the trace map from \mathbb{F}_q to \mathbb{F}_p . This extends Richman's theorem which treats the case of $f(x)$ being a monomial. Moreover, we show that the generating series $\sum_{s=1}^{\infty} N_s(f, a)t^s$ is a rational function in t and also present its explicit

expression in terms of the first $2d + 1$ initial values $N_1(f, a), \dots, N_{2d+1}(f, a)$, where d is a positive integer no more than $q - 1$. From this result, the theorems of Chowla-Cowles-Cowles and of Myerson can be derived. This is a joint work with Drs. Yulu Feng, Junyong Zhao and Chaoxi Zhu.

Speaker: 田野 (中国科学院晨兴数学中心)

Title: L -values of elliptic curves and ternary quadratic forms

Abstract: Tunnell related L -values of congruent number elliptic curves to certain ternary quadratic forms. Gross etc established such result for elliptic curves with square-free conductor case. In this talk, we introduce Tunnell-Gross type formula for general case. It is joint work with Wei He and Wei Xiong.

Speaker: 扶磊 (清华大学)

Title: A p -adic Landau-Ginzburg B-model

Abstract: The Landau-Ginzburg (LG) B-model associated to a Laurent polynomial is construct from the algebraic twisted de Rham complex using complex Hodge theory. We endow an arithmetic structure on the twisted de Rham complex and use p -adic Hodge theory to construct the LG B-model.

Speaker: 肖梁 (北京大学)

Title: 模形式 p 进斜率与 Bergdall-Pollack 幽灵猜想

Abstract: 在这个报告中, 我将介绍模形式的某种 p 进性质: 模形式 U_p 算子特征值的 p 进赋值, 又称为 p 进斜率。关于模形式 p 进斜率的研究始于 90 年代 Gouvea 和 Mazur 的大量数值计算和若干猜想。之后 Coleman-Mazur, Buzzard-Calegari, Bergdall-Pollack 逐渐从理论上完善关于 p 进斜率的这些猜想。我将报告在这个方向与刘若川、赵斌、Nha Truong 的合作, 在某些情况下证明 Bergdall-Pollack 的“幽灵猜想”。

Speaker: 欧阳毅 (中国科学技术大学)

Title: The growth of Tate-Shafarevich groups in $\mathbb{Z}/p\mathbb{Z}$ -extensions

Abstract: Let p be a prime number. Kęstutis Česnavičius proved that for an abelian variety A over a global field K , the p -Selmer group $\text{Sel}_p(A/L)$ grows unboundedly when L ranges over the $\mathbb{Z}/p\mathbb{Z}$ -extensions of K . Moreover, he

raised a further problem: is the dimension of $\text{Sha}(A/L)[p]$ also unbounded under the above conditions? In this talk we give a positive answer to this problem in the case p not equal $\text{char}K$. This result enable us to generalize the work of Clark, Sharif and Creutz on the growth of potential Sha in cyclic extensions. We also answer a problem poposed by Lim and Murty concerning the growth of the fine Tate-Shafarevich groups. This is joint work with Jianfeng Xie.

Speaker: 岳勤 (南京航空航天大学)

Title: Weil 指数和与 Kloosterman 和性质以及在编码运用

Abstract: 在这个报告中, 我们将运用离散傅立叶变换计算一下指数和, 建立 Weil 指数和与 Kloosterman 和之间连接, 由此给出一些线性码和序列的相关值。

Speaker: 向青 (南方科技大学)

Title: Constructions of m -ovoids of the Symplectic Polar Spaces

Abstract: An m -ovoid in the symplectic polar space $W(2r - 1, q)$ is a set \mathcal{M} of points such that every maximal of $W(2r - 1, q)$ meets \mathcal{M} in exactly m points. A 1-ovoid in $W(2r - 1, q)$ is simply called an ovoid. Ovoids in $W(2r - 1, q)$ (and more generally in any classical polar space) were first defined by Thas (1981). The concept of an ovoid was later generalized to that of m -ovoid by Thas (1989) and Shult/Thas (1994).

We discuss a new method for constructing m -ovoids in the symplectic polar space $W(2r - 1, q)$ from cyclotomic strongly regular graphs constructed in a paper by Brouwer, Wilson and Xiang (1999). Using this method, we obtain many new m -ovoids which can not be derived by field reduction. This talk is based on joint work with Tao Feng and Ye Wang, both of Zhejiang University.

Speaker: 徐哲峰 (西北大学)

Title: Distribution of powers in the ring \mathbb{Z}_q

Abstract: Let λ be any real numbers with $0 < \lambda \leq 1$, $q > [\frac{1}{\lambda}]$ and $m \geq 2$ be integers. In this talk, we will study the distribution of powers a^m or linear combinations of powers with integer coefficients in residue class ring \mathbb{Z}_q with $a \in [1, \lambda q]$. Some asymptotic results will be introduced.

Speaker: 翟帅 (山东大学)

Title: On the 2-part of the Birch-Swinnerton-Dyer exact formula

Abstract: In this lecture, I will present a general lower bound for the 2-adic valuation of the algebraic part of the central L -value for all the quadratic twists of any elliptic curve over the rationals, and some stronger lower bound results for certain quadratic twists of certain elliptic curves.

Speaker: 黄炳荣 (山东大学)

Title: L -函数的亚凸性界

Abstract: 我们将介绍 LL -函数的亚凸性界问题, 并重点陈述 $GL(3)$ L -函数方面的相关结果。如果时间允许, 我们将简述与徐钊合作的 $GL(3) \times GL(2)$ L -函数混合亚凸性界的证明。